Math 3B **Name:** _____
Summer Session A 2016
Challenge Problem 2 Due: 5 July 2016

Using Calculus to understand a function

What would you do if I told you to convince me (without using wolfram alpha!) that the polynomial

$$5863 - 358094x + 111192x^2 + 680600x^3$$

has a root between 0 and 1? Probably skip this week's challenge problem (or at least that's what I'd do, as that polynomial looks terrible). Instead this week we will look at a technique often used in mathematics, where instead of answering a specific hard question, you generalize it until a pattern is more visible, and it gets a bit easier.

Part 1

Let a, b, c, d be real numbers which satisfy the following equation:

$$a + \frac{b}{2} + \frac{c}{3} + \frac{d}{4} = 0$$

and consider the polynomial with them as coefficients, namely

$$f(x) = a + bx + cx^2 + dx^3$$

Convince me that f has a root in the interval $(0, 1)$.
Hint: think about the relationship of functions with their derivatives and antiderivatives. What does the mean value theorem say again?
How does this solve the question above?

Part 2

Can this be generalized? Formulate a claim like the one above, except now for a polynomial of degree N. What, if anything from the reasoning above must change to show that this polynomial also has a root in the standard unit interval?

Note: *In this problem you are trying to write up a convincing* argument *(really, a mathematical proof) instead of just doing a calculation. As such, your answer should be in full sentences / a paragraph or two.*